Solutions to JEE Main Home Practice Test - 6 | JEE - 2024

PHYSICS

SECTION-1

1.(D) Here,

Focal length of objective, $f_0 = 1cm$

Focal length of eye-piece, $f_e = 6cm$

Least distance of distinct vision, D = 25 *cm*

Tube length, L = 30 *cm*

When the image is formed at the least distance of distinct vision, the magnification is

$$m = \frac{L}{f_0}\left(1+\frac{D}{f_e}\right) = \frac{30}{1}\left(1+\frac{25}{6}\right)$$

$$= 155$$

2.(B) Heat added $= (100\ cal/s)(4\times 60s) = 24000\ cal.$

Let find temperature be $T°C$ (water)

$$24000 = 100\times 0.2\times(T+20) + 200\times 0.5\times 20 + 200\times 80 + 200\times 1\times(T-0)$$

$$\Rightarrow \quad T = 25.5°C$$

3.(D) If E_2 is small, that will cause the balance point to shift close to Q.

4.(B) $\left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{K} = \frac{1}{K}\left(\frac{Mg}{A}\right)$

But $V \propto R^3$

So, $\frac{\Delta V}{V} = 3\left(\frac{\Delta R}{R}\right)$

Thus, $\frac{\Delta R}{R} = \frac{1}{3}\left(\frac{\Delta V}{V}\right) = \frac{Mg}{3AK}$

5.(C) $K = qV$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

$$\frac{\lambda p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

$$= \sqrt{\frac{4}{1}\times\frac{2}{1}} = 2\sqrt{2}$$

6.(B) From the first data

$$\frac{95-30}{30} = k\left(\frac{95-90}{2} - \theta_0\right) \quad \text{....(i)}$$

From the second data

$$\frac{55-50}{70}=k\left(\frac{55-50}{2}-\theta_0\right) \qquad \ldots.(ii)$$

Dividing (i) and (ii) we get

$$\frac{7}{3}=\frac{92.5-\theta_0}{52.5-\theta_0}$$

$$\Rightarrow \quad \theta_0=25.5° \qquad \ldots.(iii)$$

Let the time taken in cooling from 50ºC to 45ºC is *t*, then

$$\frac{50-45}{t}=k\left(\frac{50-45}{2}-\theta_0\right) \qquad \ldots.(iv)$$

Using $\theta_0=22.5°C,$ and dividing (i) by (ii) we get

$$\frac{t}{30}=\left(\frac{92.5-22.5}{45.5-22.5}\right) \quad \Rightarrow T=84\text{ sec}$$

7.**(A)** $\beta=\frac{\lambda D}{d}$ and $\theta=\frac{d}{D}$

$$\therefore \quad \beta=\frac{\lambda}{\theta}$$

8.(C) Let a be the acceleration upwards of the two blocks

$$a=\frac{upthrust-weight}{m}$$

$$=\frac{1}{m}\left[\frac{m}{0.5\times10^3}\times10^3\times\left(g+\frac{g}{2}\right)-mg\right]$$

$$=2g$$

Acceleration of block relative to water

$$=2g-g/2=3g/2$$

$$h=\frac{1}{2}\times\frac{3g}{2}t_1^2$$

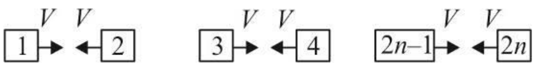
$$t=2\sqrt{\frac{h}{3g}}$$

9.(A) work done, $W=$ area under graph $=2P_0V_0$

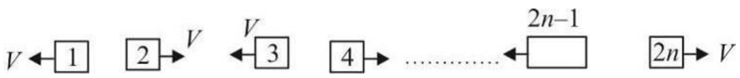
Heat supplied, $Q_{in}=Q_{AB}+Q_{BC}=\left[n\left(\frac{3R}{2}\right)\left(\frac{2P_0V_0}{nR}\right)\right]+\left[n\left(\frac{3R}{2}\right)\left(\frac{3P_0V_0}{nR}\right)+3P_0V_0\right]=\frac{21}{2}P_0V_0$

$$\eta=\frac{W}{Q_{in}}=\frac{2P_0V_0}{21P_0V_0/2}=\frac{4}{21}$$

10.(C)



After n colliosns



Next we will have $(n-1)$ collisions

This trend will continue till at the end we have only one collision.

Total no. of collisions $= 1 + 2 + 3 + ... + (n-1) + n$

$$= \frac{n(n+1)}{2}$$

11.(B) $n_1 \frac{\lambda_1}{2} = L$

and $n_2 \frac{\lambda_2}{2} = L$

from equations. (1) and (2),

$n_1 = \frac{2L}{\lambda_1}$ and $n_2 = \frac{2L}{\lambda_2}$

$$n_2 - n_1 = \frac{2L(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2}$$

12.(C) $a_t = \frac{dv}{dt} = 4t + 1 = 9$ m/s^2, $\quad a_r = \frac{v^2}{r} = \frac{100\sqrt{19}}{100} = \sqrt{19}$ m/s^2

$a_{net} = \sqrt{9^2 + (\sqrt{19})^2} = \sqrt{100} = 10$ m/s^2

13.(C)

A	B	X
1	1	1
1	0	1
0	1	1
0	0	0

OR Gate

14.(C) If w of the earth doubles, w of the satellite must also double for it to be a geostationary satellite.

$\Rightarrow$ Time period should become half

$T^2 \propto r^3$

$$\Rightarrow \frac{(T/2)^2}{T^2} = \left(\frac{R'}{R}\right)^3$$

$$\Rightarrow \quad R' = \frac{R}{4^{1/3}}$$

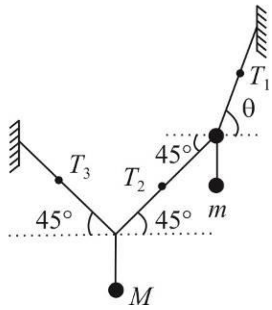
15.(A)

16.(B) $T_1 \cos\theta = T_2 \cos 45°$

$T_2 \cos 45° = T_3 \cos 45°$

$T_1 \sin\theta = T_2 \sin 45° + mg$

$T_2 \sin 45° + T_3 \sin 45° = Mg$



On solving

$$\Rightarrow T_2 = T_3 = \frac{Mg}{\sqrt{2}}$$

$$T_1 \cos\theta = \frac{Mg}{2}$$

$$T_1 \sin\theta = \frac{Mg}{2} + mg$$

$$\Rightarrow \tan\theta = \left(1 + \frac{2m}{M}\right)$$

17.(B) Energy of a charged capacitor, $E = \frac{1}{2}\frac{Q^2}{C}$

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(\frac{b}{a}\right)} \qquad \text{(for a cylindrical capacitor)}$$

$$E = \frac{1}{2}\frac{Q^2}{2\pi\varepsilon_0 L}\log_e\left(\frac{b}{a}\right) \qquad \ldots\text{(i)}$$

Where L = length of the cylinders

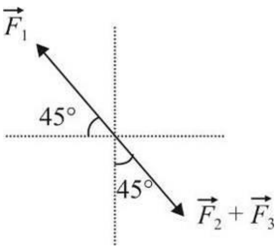
a and *b* = radii of two concentric cylinders

$$C' = \frac{2\pi\varepsilon_0 (2L)}{\log_e\left(\frac{b}{a}\right)} \; or \; E' = \frac{1}{2}\frac{(2Q)^2}{C'}$$

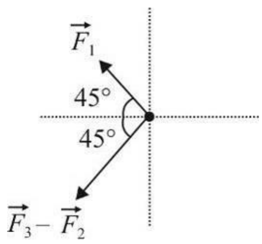
$$= \frac{1}{2}\frac{(2Q)^2}{2\pi\varepsilon_0 (2L)}\log_e\left(\frac{b}{a}\right) \qquad \ldots\text{(ii)}$$

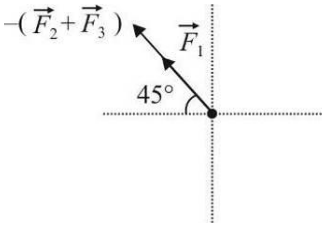
From eqns. (i) and (ii), we get; $E' = 2E$

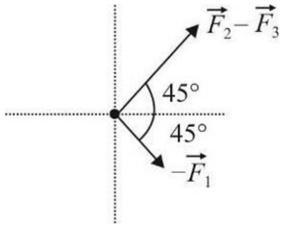
18.(B) **(A)**



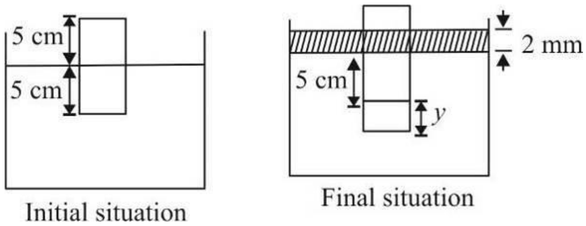
(B)



(C) 

(D) 

19.(C) Let the cube dips further by y cm and water level rises by 2 mm.



$\Rightarrow$ Volume of water raised = volume of extra depth of wood.

$$\Rightarrow \quad 100y = (1500-100)\frac{2}{10} = 1400\times\frac{2}{10} = 280$$

$\therefore$ y = 2.8 cm

$\therefore$ Extra upthrust

$$\rho_{water}\times(2.8+0.2)\times100g = mg$$

m = 300 gm

20.(C) $\dfrac{ml^2}{3}\times2+\dfrac{ml^2}{12}+m\times\left(\dfrac{\sqrt{3}l}{2}\right)^2$

$$=\frac{8ml^2+ml^2+9ml^2}{12}=\frac{3}{2}ml^2=3mk^2$$

$$k=\frac{l}{\sqrt{2}}$$

SECTION-2

1.(0) The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero.
Hence, the net field at the centre will be zero.

2.(72) $40\times2 = 10\,\alpha$ …..(i)

$$\theta=\frac{1}{2}\alpha t^2=\frac{1}{2}\times8\times3^2=36\,\text{radians}$$

$$l=r\theta=72\,m$$

3.(28) Both electric and gravitational potential energy are changing. For external point a charged sphere behaves whole of its charge is concentrated at its centre.
Apply conservation of energy for initial and final position,

$$\frac{q\times q}{4\pi\varepsilon_0\times9}+mg\times9=\frac{q^2}{4\pi\varepsilon_0(1)}+mg\times(1)$$

$$q^2=\frac{80\times10^{-3}\times9.8}{10^9}=28\mu C$$

4.(53) Acceleration at point of relase $=g\sin\theta$

Acceleration at bottom most point $=\dfrac{v^2}{R}$

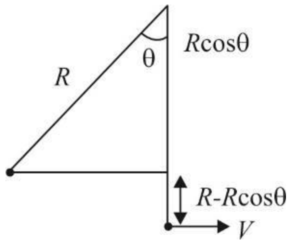
Conserving energy

$$mg\left(R-R\cos\theta\right)=\frac{1}{2}mv^2$$

$$\Rightarrow \qquad \frac{v^2}{R}=2g\left(1-\cos\theta\right)$$

$$g\sin\theta=2g\left(1-\cos\theta\right)$$

on solving, we get $\theta=53^\circ$



5.(20) $R_1=\dfrac{(200)^2}{25}=1600\,\Omega$

$$R_2=\frac{(200)^2}{100}=400\,\Omega$$

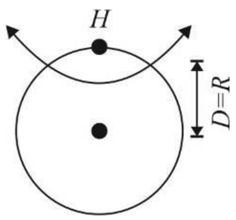
Req. = 2000 Ω

$$\text{Req}=\frac{(200)^2}{2000}=20W$$

6.(3) Time period of a physical pendulum:

$$T=2\pi\sqrt{\frac{I_0}{mgd}}=2\pi\sqrt{\frac{\left(\frac{1}{2}mR^2+mR^2\right)}{mgR}}=2\pi\sqrt{\frac{3R}{2g}} \qquad \ldots\text{(i)}$$

$$T_{simple\ pendulum}=2\pi\sqrt{\frac{l}{g}} \qquad \ldots\text{(ii)}$$



Equating (i) and (ii), $l=\dfrac{3}{2}R=3m$

7.(9) $\dfrac{1}{2}Li_0^2=\dfrac{1}{2}CV^2$

$$i_0^2=\frac{C}{L}V^2$$

$$i_0=\sqrt{C/L}\,V$$

$$i_1=\sqrt{(C/L_1)}V;\ i_2=\sqrt{(4C/L_2)}V$$

Given $i_1:i_2=1:6$

So, $L_1 = 9L_2$, $L_2 = L_1/9$

$L_1 : L_2 = L : \frac{L}{9} = 9:1$

8. (4) Avg. time period = $\frac{2.63+2.56+2.42+2.71+2.78}{5}$

$= 2.62s$

Time period	**abs error**
2.63	0.01
2.56	0.06
2.42	0.20
2.71	0.09
2.78	0.16

% error $= \frac{0.52}{5\times 2.62}\times 100 = 3.97\% = 4\%$

9.(36) $\phi = BA\cos\theta = \frac{1}{2}B\pi r^2 \cos\omega t$

$\therefore \quad e_{\text{induced}} = -\frac{d\varphi}{dt} = \frac{1}{2}B\pi r^2 \omega \sin\omega t$

$P = \frac{e^2}{R} \quad \therefore \quad P_{\text{mean}} = \frac{\int_0^T P\,dt}{T} = \frac{(B\pi r^2\omega)^2}{8R} = 36W$

10.(5) Velocity of first block before collision

$v_1^2 = 1^2 - 2(2)\times 0.16 = 1 - 0.64$

$v_1 = 0.6$ m/s

By conservation of momentum, 2 × 0.6 = $2v_1' + 4v_2'$

Also $v_2' - v_1' = v_1$ for elastic collision

It gives $v_2' = 0.4\,\text{m/s}$

$v_1' = -0.2\,\text{m/s}$

Now distance moves after collision

$s_1 = \frac{(0.4)^2}{2\times 2}$ and $s_2 = \frac{(0.2)^2}{2\times 2}$

$\therefore s = s_1 + s_2 = 0.05m = 5cm.$

CHEMISTRY

SECTION-1

1.(A) Gabriel phthalimide synthesis is for the preparation of aliphatic primary amines.

2.(D)

	Compound	**Oxidation state of meal**
(i)	CrO_5	+6
(ii)	V_2O_5	+5
(iii)	Al_2O_3	+3
(iv)	Na_2O	+1

3.(A) For isoelectronic species $\left[\frac{e}{p}\text{value } \alpha \text{ size}\right]$, so (A) is correct answer

4.(B) Six large bromide ions cannot be accommodated around Si^{4+} due to limitation of its size.

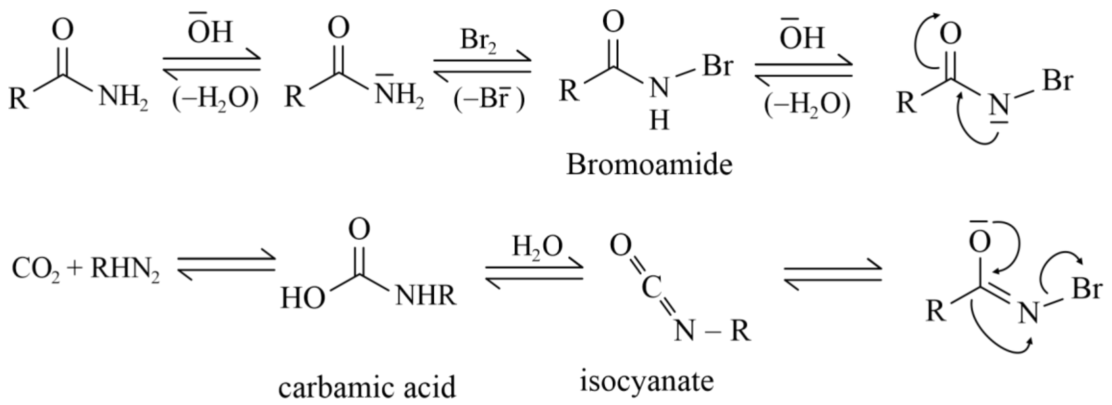
The species like, $SiF_6^{2-}, [GeCl_6]^{2-}, [Sn(OH)_6]^{2-}$ exist where the hybridisation of the central atom is sp^3d^2.

5.(A) $Fe^{3+}; 3d^5 = t_{2g}^5 e_g^0$

It will contain 1 unpaired electrons.
Thus it is paramagnetic and active in external magnetic field.

6.(C)

7.(D)



8.(D) $A_t = A_0 - kt;$ Zero order

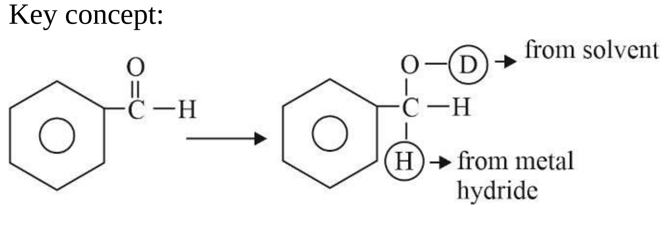
$t_{1/2} = \left(\frac{A_0}{2k}\right)$

9.(D) It is globular protein (water soluble)

10.(A) Factual

11.(D) Grignard reagent act as strong base.

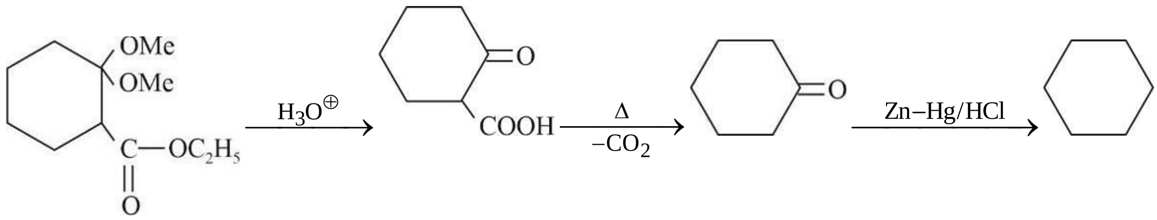
12.(C) Key concept:



13.(A) Key concept: any compound which is more acidic than H_2CO_3 releases CO_2 on reaction with $NaHCO_3$.

14.(A) As it is linear polymer of $\alpha - D$ glucose, more number of $-OH$ groups remain free for $H-$bonding with water.

15.(D)

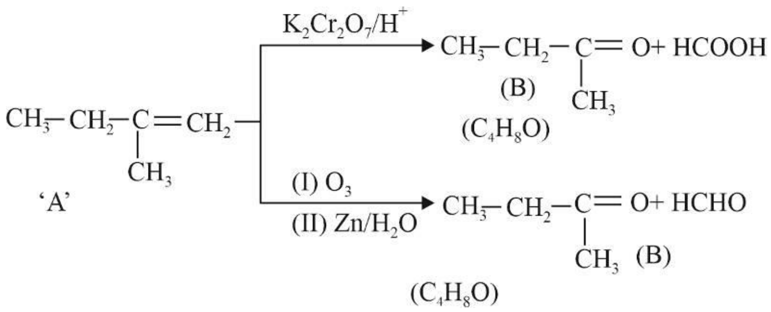


16.(C) **Keypoint:-** Only one degree aliphatic and aromatic amines give carbyl amine test due to presence of two hydrogen connected to nitrogen atom and form isocyanide.

17.(D) Compounds having only one chiral centre have only enantiomeric isomers

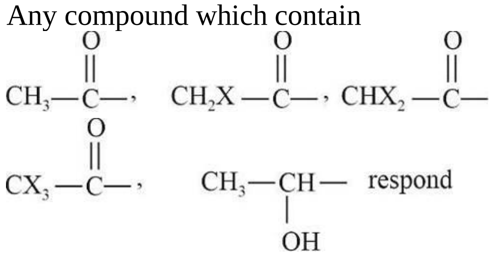
18.(A) LDA abstracts more acidic $-H$.

19.(A)



20.(C)

Any compound which contain



, here $X = I$

to Iodoform test.

SECTION-2

1.(330) $C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow nCO_2 + (n+1)H_2O$

$\because$ 1mole C_3H_8 produces 4 mole H_2O.

4 mole$(4\times18) = 72$gm

$72\text{gm} \rightarrow 44\text{g of } C_3H_8$

So $54gH_2O \rightarrow \frac{44\times54}{72}$

$= 33$

2.(48)

$$
\begin{array}{llll}
 & X+ & Y \rightleftharpoons & 2Z \quad K_C = 64 \\
t=0 & 2 & 2 & 2 \\
t=t_{eq.} & 2-x & 2-x & 2+2x
\end{array}
$$

$$K_C = \left(\frac{2+2x}{2-x}\right)^2$$

$$64 = \left(\frac{(2+2x)}{(2-x)}\right)^2$$

So $8 = \left(\frac{2+2x}{2-x}\right) \Rightarrow x = \left(\frac{7}{5}\right)$

$\therefore$ Concentration of $[Z] = 2 + 2 \times \frac{7}{5} = 48 \times 10^{-1}$

3.(9) $Cr_2O_7^{2-} + Sn^{2+} \to Sn^{4+} + Cr^{3+}$

30ml 10ml

$N_1V_1 = N_2V_2$

$30 \times 1 \times 6 = 10 \times M \times 2$

$M = 9$

4.(1) $\left[Cr(NH_3)_3(H_2O)_2 Br\right] Br_2$

5.(2) $\Delta H_{fusion} = \Delta H_{sub} - \Delta H_{vab}$

$= 101 - 99 = 2\ \text{kcal mol}^{-}$

6.(16)

7.(2) According to question

Energy per second = $\frac{400}{4} = 100J$

$E = n \times \frac{hc}{\lambda}$

$$100J = \frac{n \times 6.626 \times 10^{-34} Js^{-1} \times 3 \times 10^8 ms^{-1}}{400 \times 10^{-9} m}$$

$n = 2 \times 10^{20}$

So $x = 2$

8.(41) $\because P = 2atm$

$T = 350K$

$V = 2.0 \times 10^6 lit$

From $PV = nRT$

$$n = \left(\frac{PV}{RT}\right)$$

$$n = \frac{2 \times 2 \times 10^6}{0.083 \times 350}$$

$n = 0.1376 \times 10^6$

So mass of $C_2H_6 = n \times (M_0)_{C_2H_6}$

$= 0.1376 \times 10^6 \times 30g$

$= 4.1308 \times 10^6 g$

$= 41.308 \times 10^5 \text{g}$

So $x = 41$

9.(25) $P_Q = K_H.X_Q; X_Q = \dfrac{P_Q}{K_H} = \dfrac{0.835}{1.67 \times 10^3} = 0.5 \times 10^{-3}$

No. of moles $H_2O = \dfrac{900}{18} = 50$

$$\frac{n_Q}{n_Q + n_{H_2O}} = 0.5 \times 10^{-3}$$

$$\frac{n_Q}{n_{H_2O}} = 0.5 \times 10^{-3}$$

$n_Q = 0.5 \times 10^{-3} \times 50 = 25 \times 10^{-3}$ moles $= 25$ m moles

10.(3) $X + 2Y^{3+} \longrightarrow X^{2+} + 2Y^{2+}$

So $E^0_{cell} = E^0_R - E^0_L$

$= 0.77 - (-0.76)$

$E^0_{cell} = 1.53\text{V}$

From Nernst equation

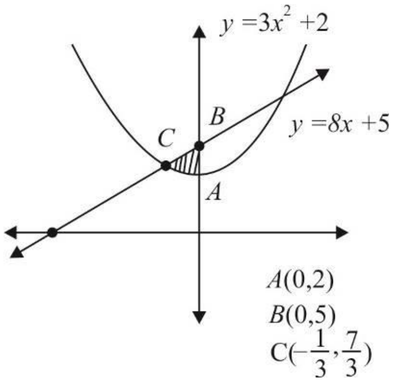
$$E_{cell} = E^0_{cell} - \frac{0.0591}{n} \log \frac{[X^{2+}][Y^{2+}]^2}{[Y^{3+}]^2}$$

So $\dfrac{[Y^{2+}]}{[Y^{3+}]} = \sqrt{10} \sim 3$

MATHEMATICS

SECTION-1

1.(C)



$$3x^2+2=8x+5$$
$$3x^2-8x-3=0$$
$$x=-1/3,3$$

Required area = $\int_{x=\frac{-1}{3}}^{0}\left[(8x+5)-(3x^2+2)\right]dx=\frac{14}{27}$

2.(D) Given $\frac{(x+3)^2}{16}-\frac{(y+2)^2}{25}=1$

Let $x+3=X$

$y+2=Y$

$$\frac{X^2}{16}-\frac{Y^2}{25}=1$$

$a=4,\ b=5$

$b^2=a^2\left(e^2-1\right)\Rightarrow e=\frac{\sqrt{41}}{4}$

Focus $(\pm ae,0)\Rightarrow X=\pm ae,\ \ Y=0$

$x+3=\pm\sqrt{41},\ y+2=0$

$x=\pm\sqrt{41}-3,\ \ y=-2$

Hence focus $S\left(\sqrt{41}-3,-2\right),\ S'\left(-\sqrt{41}-3,-2\right)$

Let any point on hyperbola $x+3=4\sec\theta, y+2=5\tan\theta\Rightarrow P\left(-3+4\sec\theta,-2+5\tan\theta\right)$

Hence centroid is $\equiv\left(\frac{-9+4\sec\theta}{3},\frac{-6+5\tan\theta}{3}\right)$

$h=\frac{-9+4\sec\theta}{3}\Rightarrow\sec\theta=\frac{3h+9}{4}$

& $\tan\theta=\frac{3k+6}{5}$

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \frac{9}{16}\left(h^2+6h+9\right) - \frac{9}{25}\left[k^2+4k+4\right] = 1$

$\Rightarrow \quad 225h^2 - 144k^2 + 1350h - 576k + 1049 = 0$

3.(B) $\frac{S_{5n}}{S_{2n}} = \frac{\frac{5n}{2}\left[2a+(5n-1)d\right]}{\frac{2n}{2}\left[2a+(2n-1)d\right]} = 4$

$\Rightarrow \quad 10a + 5(5n-1)d = 8\left(2a+(2n-1)d\right)$

$\Rightarrow \quad 6a + 16nd - 8d - 25nd + 5d = 0$

$\Rightarrow \quad 2a = (3n+1)d$

Now, $\frac{S_{3n}}{S_{2n}} = \frac{\frac{3n}{2}\left[2a+(3n-1)d\right]}{\frac{2n}{2}\left[2a+(2n-1)d\right]} = \frac{3}{2}\left[\frac{6nd}{5nd}\right] = \frac{18}{10} = \frac{9}{5}$

4.(C) The number of ways of distributing 11 distinct balls in 5 boxes B_1, B_2, B_3, B_4 and B_5, is 5^{11}

When box B_2 contains exactly 2 balls then number of ways = ${}^{11}C_2 \times 4^9$

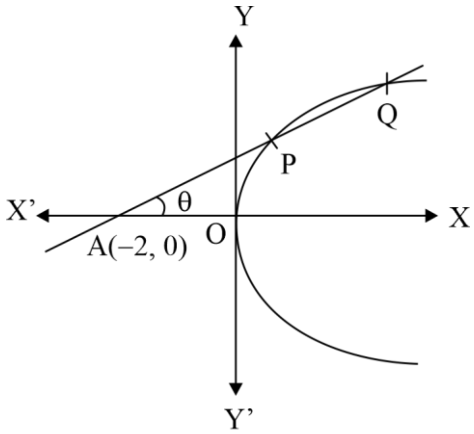
Probability $= {}^{11}C_2 \times \frac{4^9}{5^{11}} = \frac{55}{16}\left(\frac{4}{5}\right)^{11}$

Hence $k = \frac{55}{16} = 3\frac{7}{16}$

Clearly $2 < k < 4 \Rightarrow k \in \{x \in R : |x-3| < 1\}$

5.(A) Let $P(-2+r\cos\theta,\ r\sin\theta)$ and Q lie on the parabola.

Then,



$r^2\sin^2\theta - 4(-2+r\cos\theta) = 0$

Or $r_1 + r_2 = \frac{4\cos\theta}{\sin^2\theta},\ r_1r_2 = \frac{8}{\sin^2\theta}$

Now, $\frac{1}{AP} + \frac{1}{AQ} = \frac{r_1+r_2}{r_1r_2} = \frac{\cos\theta}{2}$

Given that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$

Or $\cos\theta < \frac{1}{2}$

Or $\tan\theta > \sqrt{3}$ [cos θ is decreasing and tan θ is increasing in (0, π/2)]

$\therefore m > \sqrt{3}$

6.(A) $\Delta = 0$ and any one of Δ_1, Δ_2 and Δ_3 should not be equal to zero

$$\Delta = \begin{vmatrix} 3 & -4 & 8 \\ 2 & a & -1 \\ 3 & 1 & -2 \end{vmatrix} = 0 \Rightarrow \Delta = 3(-2a+1)+4(-4+3)+8(2-3a)=0$$

$\Rightarrow a = \frac{1}{2}$

$$\Delta_1 = \begin{vmatrix} 5 & -4 & 8 \\ 2 & 1/2 & -1 \\ b & 1 & -2 \end{vmatrix} = 19b - 55$$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & 8 \\ 2 & 2 & -1 \\ 3 & b & -2 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 3 & -4 & 5 \\ 2 & 1/2 & 2 \\ 3 & 1 & b \end{vmatrix} = \frac{19b-55}{2}$$

If $b = \frac{55}{19}$, then Δ_1, Δ_2, and Δ_3 all will be zero.

$\therefore\ b \neq \frac{55}{19}$

7.(A) $RHL = \lim\limits_{x\to 1^+} \frac{\delta\left|(x-1)(x-4)\right|}{(x-1)(x-4)} = -\delta$

$LHL = \lim\limits_{x\to 1^-}\left(\frac{-\lambda\sin(1-x)}{[x-1](1-x)}\right) + 2 = \lambda + 2$

Also, $f(1) = 3\lambda$

For $f(x)$ to be continous at $x = 1$,

$RHL = LHL = f(1)$

$\therefore\ -\delta = \lambda + 2 = 3\lambda$

$\Rightarrow \quad \lambda = 1, \delta = -3$

$\Rightarrow \quad \lambda + \delta = -2$

8.(B) Let $p = \log_2 5 - \sum\limits_{k=1}^{4} \log_2\left(\sin\left(\frac{k\pi}{5}\right)\right)$

$$= \log_2 5 - \left\{\log_2\left(\sin\left(\frac{\pi}{5}\right)\right) + \log_2\left(\sin\left(\frac{2\pi}{5}\right)\right) + \log_2\left(\sin\left(\frac{3\pi}{5}\right)\right) + \log_2\left(\sin\left(\frac{4\pi}{5}\right)\right)\right\}$$

$= \log_2 5 - \log_2 \left\{ \sin\frac{\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin\frac{3\pi}{5} \cdot \sin\frac{4\pi}{5} \right\}$

$= \log_2 5 - \log_2 \left\{ \sin^2\left(\frac{\pi}{5}\right) \cdot \sin^2\left(\frac{2\pi}{5}\right) \right\}$

$= \log_2 5 - \log_2 \left\{ \frac{(1-\cos 72°)(1-\cos 144°)}{4} \right\}$

$= \log_2 5 - \log_2 \left\{ \frac{(1-\sin 18°)(1+\cos 36°)}{4} \right\}$

$= \log_2 5 - \log_2 \left\{ \frac{\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)}{4} \right\}$

$= \log_2 5 - \log_2 \left\{ \frac{\left(5-\sqrt{5}\right)\left(5+\sqrt{5}\right)}{64} \right\} = \log_2 2^4 = \frac{4}{1} = \frac{p}{q}$

$= \log_2 \left(5 \times \frac{16}{5}\right) = \log_2 2^4 = \frac{4}{1} = \frac{p}{q}$ (given)

$\therefore \quad p = 4,\ q = 1$

Hence, $p^2 + q^2 = 4^2 + 1^2 = 17$

9.(C) Given circles are

$(x - x_0)^2 + (y - y_0)^2 = r^2$

And $(x - x_0)^2 + (y - y_0)^2 = 4r^2$

Or $|z - z_0| = r$ ….(i)

And $|z - z_0| = 2r$ ….(ii)

Where $z_0 = x_0 + iy_0$

Now α and $\frac{1}{\alpha}$ lies on circles (i) and (ii), respectively. Then

$|\alpha - z_0| = r$ & $\left|\frac{1}{\alpha} - z_0\right| = 2r$

$\Rightarrow \quad |\alpha - z_0| = r$ & $|1 - \bar{\alpha} z_0| = 2r\, |\vec{\alpha}|$

$\Rightarrow \quad |\alpha - z_0|^2 = r^2$ & $|1 - \bar{\alpha} z_0|^2 = 4r\, |\alpha|^2$

Subtracting, we get

$|1 - \bar{\alpha} z_0|^2 - |\alpha - z_0|^2 = 4r^2 |\alpha|^2 - r^2$

$\Rightarrow \quad 1 + |\alpha z_0|^2 - \bar{\alpha} z_0 - \alpha \bar{z}_0 - \left(|\alpha|^2 + |z_0|^2 + \bar{\alpha} z_0 - \alpha \bar{z}_0\right)$

$= 4r^2 |\alpha|^2 - r^2$

$\Rightarrow \quad 1 + |\alpha|^2 |z_0|^2 - |\alpha|^2 - |z_0|^2 = 4r^2 |\alpha|^2 - r^2$

$\Rightarrow \quad \left(1-|\alpha|^2\right)\left(1-|z_0|^2\right)=4r^2|\alpha|^2-r^2$

Given $2|z_0|^2=r^2+2$

$\Rightarrow \quad \left(1-|\alpha|^2\right)\left(1-\frac{r^2+2}{2}\right)=4r^2|\alpha|^2-r^2$

$\Rightarrow \quad \left(1-|\alpha|^2\right)\left(\frac{-r^2}{2}\right)=4r^2|\alpha|^2-r^2$

$\Rightarrow \quad |\alpha|^2-1=8|\alpha|^2-2$

$\Rightarrow \quad |\alpha|^2=\frac{1}{7} \qquad \Rightarrow |\alpha|=\frac{1}{\sqrt{7}}$

10.(C) $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$$I=\int_{\pi/36}^{5\pi/36}\frac{(\sec 3x)^{1/5}dx}{(\operatorname{cosec}3x)^{1/5}+(\sec 3x)^{1/5}} \qquad \text{.....(i)}$$

By property

$$I=\int_{\pi/36}^{5\pi/36}\frac{(\operatorname{cosec}3x)^{1/5}}{(\sec 3x)^{1/5}+(\operatorname{cosec}3x)^{1/5}} \qquad \text{....(ii)}$$

By adding (i) & (ii)

$$2I=\int_{\pi/36}^{5\pi/36}dx=\frac{\pi}{9}\Rightarrow I=\frac{\pi}{18}$$

11.(B) Three vectors are coplanar

The given determinant

$$\Rightarrow \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}=0$$

$$\Rightarrow \quad \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}=0$$

$(C_1 \to C_1+C_2+C_3)$

$$\Rightarrow \quad 2(a+b+c)\times\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}=0$$

$$\Rightarrow \quad 2(a+b+c)\times\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}=0$$

$\begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{pmatrix}$

$\Rightarrow \quad 2(a+b+c)^3 \times \begin{vmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad 2(a+b+c)^3 \times 1 = 0$

$\Rightarrow \quad 2(a+b+c)^3 \times 1 = 0$

$\Rightarrow \quad a+b+c=0$

12.(D) $g(\alpha)=\beta$

$g(\beta)=\gamma$

$g(\gamma)=\alpha$

For $x=\alpha$

(1) $gogog(\alpha)=gog(\beta)=g(\gamma)\neq g(\alpha)$

$\therefore$ $gogog \neq g$

(2) As $f: N \to N$,

Let $g(f(x))=f(x)$

$\Rightarrow \quad g(\alpha)=\alpha$ {Let $\alpha = f(x)$}

$\Rightarrow \quad \beta=\alpha$ which is not necessarily true

(3) Let $f: N \to N$ such that $f(g(x))=f(x)$ is one-one f^n.

$\Rightarrow \quad g(x)=x$ which is false.

13.(A) Let the equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$

It passes through $\left(2, \frac{2\sqrt{5}}{3}\right) \Rightarrow \frac{4}{a^2}+\frac{20}{9b^2}=1$ **......(i)**

Given $e=\frac{\sqrt{5}}{3} \Rightarrow b^2=a^2\left(1-e^2\right)=\frac{4a^2}{9}$ ...(2)

Solving (1) & (2) we get $a^2=9, b^2=4$

$\therefore$ Ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$ ....(3)

Focus $(\pm ae, 0) \equiv \left(\pm\sqrt{5}, 0\right)$

Hence, circle is $\left(x-\sqrt{5}\right)^2+y^2=\frac{101+36\sqrt{5}}{9}$ ...(4)

Solving (3) & (4) $\Rightarrow \quad x=-2;\ y=\pm\frac{2\sqrt{5}}{3}$

$\therefore \quad P\left(-2, \dfrac{2\sqrt{5}}{3}\right) \;\&\; Q\left(-2, -\dfrac{2\sqrt{5}}{3}\right)$

$PQ = \dfrac{4\sqrt{5}}{3} \Rightarrow PQ^2 = \dfrac{80}{9}$

14.(B) $f(x) = \begin{cases} -1+x, & -\infty < x < 0 \\ -1+\sin x, & 0 \le x < \pi/2 \\ \cos x, & \pi/2 \le x < \infty \end{cases}$

$f'(x) = \begin{cases} 1, & -\infty < x < 0 \\ \cos x, & 0 < x < \pi/2 \\ -\sin x, & \pi/2 < x < \infty \end{cases}$

$f'(0^-) = 1, f'(0^+) = 1$ and $f(x)$ is continuous at $x = 0$

$\Rightarrow \quad f(x)$ is differentiable at $x = 0$

$f'\left(\dfrac{\pi^-}{2}\right) = 0, \quad f'(\pi/2^+) = -1$

$\Rightarrow \quad f(x)$ is not differentiable at $x = \dfrac{\pi}{2}$.

$\therefore$ Number of points of non-differentiability is 1.

15.(D) $f(x) = \dfrac{3}{2}\sin^4 x + (4+\sqrt{3})\sin^3 x + 3\sqrt{3}\sin^2 x - 2$

$f'(x) = 6\sin^3 x\cos x + (12+3\sqrt{3})\sin^2 x\cos x + 6\sqrt{3}\sin x\cos x$

$= 3\sin x\cos x\left(2\sin^2 x + (4+\sqrt{3})\sin x + 2\sqrt{3}\right)$

$= 3\sin x\cos x\left(2\sin x + \sqrt{3}\right)(\sin x + 2)$

$f'(x) = \dfrac{3}{2}\sin 2x\left(2\sin x + \sqrt{3}\right)(\sin x + 2)$

$f'(x) = 0 \Rightarrow x = 0, \dfrac{-\pi}{3}$

Decreasing in $x \in \left(\dfrac{-\pi}{3}, 0\right)$

16.(C) $e^{8x} - 2e^{6x} + 4e^{4x} - 4e^{2x} - 3e^{4x} + 4 = 0$

$\Rightarrow \quad (e^{4x}+2)^2 - 2(e^{4x}+2)(e^{2x}) - 3e^{4x} = 0$

$\Rightarrow \quad m^2 - 2mn - 3n^2 = 0$

$\Rightarrow \quad (m-3n)(m+n) = 0$

$\Rightarrow \quad m = 3n \quad$ or $\quad m + n = 0$

$\Rightarrow \quad e^{4x} + 2 = 3e^{2x}$ or $e^{4x} + 2 + e^{2x} = 0$

$\Rightarrow \quad (e^{2x}-1)(e^{2x}-2)=0$ or $x\in\phi$

$\Rightarrow \quad e^{2x}=1$ or $e^{2x}=2$

$\Rightarrow \quad x=0$ or $x=\frac{1}{2}\ln(2)$

17.(C) $\frac{dy}{dx}=3-2xe^{6x-2y}$ …(1)

$e^{2y}\frac{dy}{dx}=3e^{2y}-2xe^{6x}$

Put $e^{2y}=t\Rightarrow 2e^{2y}\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{1}{2}\frac{dt}{dx}=3t-2xe^{x}\Rightarrow\frac{dt}{dx}+(-6t)=-4xe^{6x}$ …(2)

I.F. $=e^{\int -6dx}=e^{-6x}$

Solution of equation (2) is

$te^{-6x}=\int\left(-4xe^{6x}\right)e^{-6x}dx+c$

$e^{2y-6x}=-2x^2+c$ ….(3)

$y(0)=0\Rightarrow 1=c$

$(3)\Rightarrow e^{2y-6x}=1-2x^2$

$\Rightarrow \quad 2y-6x=\ln\left(1-2x^2\right)\Rightarrow y=3x+\frac{1}{2}\ln\left(1-2x^2\right)$

$\therefore \quad y|_{x=1/2}=\frac{3}{2}+\frac{1}{2}\ln\left(\frac{1}{2}\right)=\frac{3}{2}-\frac{1}{2}\ln(2)$

$\Rightarrow \quad 2y\left(\frac{1}{2}\right)+\ln(2)=3$

18.(D) $\frac{1}{a+b}+\frac{1}{a+2b}+\frac{1}{a+3b}+\ldots\ldots\ldots\ldots+\frac{1}{a+nb}$

$=\frac{1}{a}\left[\left(1+\frac{b}{a}\right)^{-1}+\left(1+\frac{2b}{a}\right)^{-1}+\left(1+\frac{3b}{a}\right)^{-1}+\ldots\ldots+\left(1+\frac{nb}{a}\right)^{-1}\right]$

$=\frac{1}{a}\left[\left\{1-\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2-\ldots\ldots\right\}+\left\{1-\left(\frac{2b}{a}\right)+\left(\frac{2b}{a}\right)^2-\ldots\right\}+\ldots\ldots+\left\{1-\left(\frac{nb}{a}\right)+\left(\frac{nb}{a}\right)^2-\ldots\ldots\right\}\right]$

$=\frac{1}{a}\left[n-\frac{b}{a}(1+2+\ldots\ldots n)+\frac{b^2}{a^2}\left(1^2+2^2+\ldots\ldots+n^2\right)\right]$

$=\frac{1}{a}\left[n-\frac{n(n+1)}{2}\frac{b}{a}+\frac{n(n+1)(2n+1)}{6}\frac{b^2}{a^2}\right]$

$=\frac{1}{a}\left[n-\frac{n^2}{2}\frac{b}{a}-\frac{n}{2}\frac{b}{a}+\frac{2n^3+3n^2+n}{6}\left(\frac{b^2}{a^2}\right)\right]$

$=n\left(\frac{1}{a}-\frac{b}{2a^2}+\frac{b^2}{6a^3}\right)+\left(\frac{-b}{2a^2}+\frac{b^2}{2a^3}\right)n^2+\frac{b^2}{3a^3}n^3$

By comparing this result to $\alpha n+\beta n^2+\gamma n^3$

we get $\alpha-\beta=\frac{1}{a}-\frac{b^2}{3a^3}=\frac{3a^2-b^2}{3a^3}$

19.(B) The number of trains a day (the digits 1, 2, 3) are three groups of like elements from which a sample must be formed. In the time-table for a week, the number 1 is repeated twice, the number 2 is repeated 3 times, and the number 3 is repeated twice.

The number of different time-tables is given by

$$p(2,\ 3,\ 2)=\frac{7!}{2!3!2!}=210$$

20.(C) $a\hat{i}+b\hat{j}+\frac{1}{2}\hat{k}=l(\hat{i}+2\hat{j})+m\left(\hat{j}-2\hat{k}\right)$

$\Rightarrow \quad a=l,\ b=2l+m\ \ \&\ \ m=\frac{-1}{4}$

$a\hat{i}+b\hat{j}+\frac{1}{2}\hat{k}$ is unit vector

$\therefore \quad a^2+b^2=\frac{3}{4} \quad \Rightarrow 5a^2-a-\frac{11}{16}=0$

a_1 and a_2 are roots of above equation

$\Rightarrow \quad \frac{1}{a_1}+\frac{1}{a_2}=\frac{a_1+a_2}{a_1a_2}=\frac{16}{11}$

SECTION-2

1.(4) Let $x=\frac{1}{2}+\frac{4}{2^2}+\frac{7}{2^3}+\frac{10}{2^4}+........\infty$ …..(1)

$\frac{1}{2}x=\frac{1}{2^2}+\frac{4}{2^3}+\frac{7}{2^4}+\frac{10}{2^5}+........\infty$ **…(2)**

(1)-(2) $\Rightarrow \quad \frac{x}{2}=\frac{1}{2}+\left[3\left(\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+.........\infty\right)\right]$

$\Rightarrow \quad \frac{x}{2}=\frac{1}{2}+3\left[\frac{1/4}{1-1/2}\right]=\frac{1}{2}+\frac{3}{2}=2$

$\Rightarrow \quad x=4$

Also, Let $y=\frac{2}{5}+\frac{2^2}{5^2}+\frac{2^3}{5^3}+.......\infty$

$y=\frac{2/5}{1-2/5}=\frac{2}{3}$

Now, $\left(\frac{1}{2}+\frac{4}{2^2}+\frac{7}{2^3}+.....\infty\right)^{\log_{\frac{2}{3}}\left(\frac{2}{5}+\frac{2^2}{5^2}+\frac{2^3}{5^3}+.........\infty\right)}$

$=\left(x\right)^{\log_{\frac{2}{3}}y}=\left(4\right)^{\log_{\frac{2}{3}}(2/3)}=4^1=4$

2.(8)

Class	10^{th}	11^{th}	12^{th}
No. of students	6	7	8

3	3	$6 \rightarrow {}^6C_3 \times {}^7C_3 \times {}^8C_6$
3	4	$5 \rightarrow {}^6C_3 \times {}^7C_4 \times {}^8C_5$
4	3	$5 \rightarrow {}^6C_4 \times {}^7C_3 \times {}^8C_5$

Total no. of ways = ${}^7C_3\left[{}^6C_3\left({}^8C_6+{}^8C_5\right)+{}^6C_4\times{}^8C_5\right]=88200=10000k$

$\Rightarrow \quad k=8.82 \Rightarrow [k]=8$

3.(2) $3\alpha^2-19\sqrt{5}\alpha=-7^{1/4}$

& $3\beta^2-19\sqrt{5}\beta=-7^{1/4}$

Now, $\dfrac{P_{13}\left(P_{18}-19\sqrt{5}P_{16}\right)}{P_{16}\left(P_{15}-19\sqrt{5}P_{13}\right)}=\dfrac{P_{13}\left(\alpha^{18}-\beta^{18}-19\sqrt{5}\left(\alpha^{16}-\beta^{16}\right)\right)}{P_{16}\left(\alpha^{15}-\beta^{15}-19\sqrt{5}\left(\alpha^{13}-\beta^{13}\right)\right)}$

$=\dfrac{P_{13}\left(\alpha^{16}\left(\alpha^2-19\sqrt{5}\right)-\beta^{16}\left(\beta^2-19\sqrt{5}\right)\right)}{P_{16}\left(\alpha^{13}\left(\alpha^2-19\sqrt{5}\right)-\beta^{13}\left(\beta^2-19\sqrt{5}\right)\right)}=\dfrac{P_{13}\left(\alpha^{16}-\beta^{16}\right)\left(-7^{1/4}\right)}{P_{16}\left(\alpha^{13}-\beta^{13}\right)(-7)^{1/4}}$

$=\dfrac{P_{13}P_{16}}{P_{16}P_{13}}=1$

Also, $\left(\dfrac{P_2}{\alpha-\beta}\right)\left(\dfrac{3}{19\sqrt{5}}\right)=\left(\dfrac{\alpha^2-\beta^2}{\alpha-\beta}\right)\dfrac{3}{19\sqrt{5}}=1\left\{\because \alpha+\beta=\dfrac{19\sqrt{5}}{3}\right\}$

$\therefore \quad \text{Ans}=1+1=2$

4.(2) $f(x)=\displaystyle\int\frac{x^{2009}}{\left(1+x^2\right)^{1006}}dx$

Put $1+x^2=t \qquad \Rightarrow 2x\,dx=dt$

$\therefore \qquad I=\dfrac{1}{2}\displaystyle\int\frac{(t-1)^{1004}\,dt}{t^{1006}}=\frac{1}{2}\int\left(1-\frac{1}{t}\right)^{1004}\cdot\frac{1}{t^2}dt$

Put $1-\dfrac{1}{t}=y \qquad \Rightarrow \dfrac{1}{t^2}dt=dy$

$\Rightarrow \qquad I=\dfrac{1}{2}\displaystyle\int y^{1004}dy=\frac{1}{2}\cdot\frac{y^{1005}}{1005}+C$

$=\dfrac{1}{2010}\cdot\left(\dfrac{t-1}{t}\right)^{1005}+C=\dfrac{1}{2010}\cdot\left(\dfrac{x^2}{1+x^2}\right)^{1005}+C$

$\Rightarrow m=1005,\ n=2010 \Rightarrow \dfrac{n}{m}=\dfrac{2010}{1005}=2$

5.(9) $\vec{p}=\hat{i}+2\hat{j}-\hat{k}$

$2\vec{p}-3\vec{q}=-4\hat{i}-5\hat{j}+\hat{k}$

$$\vec{p}\times\left(2\vec{p}-3\vec{q}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ -4 & -5 & 1\end{vmatrix}=-3\hat{i}+3\hat{j}+3\hat{k}$$

$$\vec{r}=\pm 3\sqrt{3}\left(\frac{-3\hat{i}+3\hat{j}+3\hat{k}}{\left|-3\hat{i}+3\hat{j}+3\hat{k}\right|}\right)=\pm\left(-3\hat{i}+3\hat{j}+3\hat{k}\right)$$

$\therefore\ |\alpha|+|\beta|+|\gamma|=9$

6.(6) Let $A=\begin{bmatrix}0 & 1\\ i & 0\end{bmatrix}^n$ and $B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$AB=IB$

$(A-I)B=0$

$A=I$

$\begin{bmatrix}0 & 1\\ i & 0\end{bmatrix}^n=1$

$A^8=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$n=$ multiple of 8

Number of three digit numbers in $S=112$ $\quad (104,112,120,\ldots\ldots\ldots\ldots,992)\Rightarrow\lambda_1=112$

Number of one digit numbers in $S=2\{0\ \&8\}\Rightarrow\lambda_2=2$. Hence, $\lambda_1+\lambda_2=114$

$\Rightarrow 19\lambda=112+2=114\Rightarrow\lambda=6$

7.(1)

$$\frac{{}^{15}C_7+{}^{15}C_8}{{}^{16}C_8}=\frac{{}^{16}C_8}{{}^{16}C_8}=1$$

8.(3) $$\left[3^{2\log_3\left(3+x^{1/3}\right)^{1/2}}-4^{\log_4\left(\frac{3\sqrt{x}+1}{\sqrt{x}}\right)^{(2/3)\left(\frac{3}{2}\right)}}\right]^{10}$$

$$=\left[\left(x^{1/3}+3\right)-\left(\frac{3\sqrt{x}+1}{\sqrt{x}}\right)\right]^{10}$$

$$=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

$$T_{r+1}={}^{10}C_r\left(x^{1/3}\right)^{10-r}\left(-x^{-1/2}\right)^r$$

$$\frac{10-r}{3}-\frac{r}{2}=0\Rightarrow 20-2r-3r=0$$

$\Rightarrow\quad r=4$

$T_5={}^{10}C_4=\dfrac{10\times9\times8\times7}{4\times3\times2\times1}=210\Rightarrow$ sum of digits $=3$

9.(3) $|A| = (ad - bc) = 10$

where $a, b, c, d \in \{\pm 1, \pm 2, \pm 3\}$

Case-1 $\text{ad} = 9$ & $bc = -1$

$ad \equiv (3,3)\, or\ (-3,-3) \,\&\, bc \equiv (1,-1), (-1,1)$

Total $= 2 \times 2 = 4$ matrices

Case-II $ad = 1$ & $bc = -9$

Similarly, Total $= 2 \times 2 = 4$ matrices

Case-III $ad = 6 \,\&\, bc = -4$

$ad \equiv (2,3), (3,2), (-2,-3), (-3,-2) \,\&\, bc = (2,-2), (-2,2)$

Total $= 4 \times 2 = 8$

Case IV $ad = 4$ & $bc = -6$

$ad \equiv (2,2), (-2,-2) \,\&\, bc = (2,-3), (-2,3)(3,-2), (-3,2)$

Total $= 2 \times 4 = 8$

Hence, Required number of $A = 4 + 4 + 8 + 8 = 24 = 8\lambda \Rightarrow \lambda = 3$

10.(4)

Class	**Frequency**	**C.F.**
$10-20$	50	50
$20-30$	p	$50+p$
$30-40$	20	$70+p$
$40-50$	114	$184+p$
$50-60$	q	$184+p+q$

$N = \sum f = 534$

$p + q = 350$

Median $(m) = l + \left[\dfrac{\left(\dfrac{N}{2}\right) - c}{f}\right] \times h$

$N = \dfrac{534}{2} = 267$

$m = 32 = 30 + \left(\dfrac{267 - (50 + p)}{20}\right) 10 \Rightarrow p = 213$

$\Rightarrow \quad q = 350 - 213 = 137$

$\Rightarrow \quad \dfrac{p-q}{19} = \dfrac{76}{19} = 4$